Chapter 3

Algebra

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Algebra is a huge part of mathematics. A lot of students hear about it a long time before they actually have to do any algebra. It's not very hard to learn how to do, especially if you get to understand the basics early on. And once you know how to 'do algebra', you will be able to solve a large variety of challenging mathematics problems easily.

SECTION 3.1 - INTRODUCTION TO ALGEBRA

The first and biggest thing to understand is what a *variable* is in algebra. Say I have the following equation:

$$2+7=9$$

This is a really simple equation – everyone knows that "two plus seven equals nine". This equation has three numbers in it – a '2', a '7' and a '9'. It also has two operations – a '+' sign and an '=' sign.

Now, instead of one of these numbers, let's write a *letter* in its place:

$$\Rightarrow 2+7=9$$

$$\Rightarrow x+7=9$$

Notice how now, instead of the number '2', there is a letter 'x'. If you read this new equation out aloud, you should say something like: "x plus seven equals nine". Notice that instead of saying 'two' we now say 'x'. 'x' is what we call a *variable*, or *pronumeral*. Variables are used to *represent* numbers.

So in this case, we know that the 'x' represents the number '2', since we *replaced* the '2' with the 'x'. So if we wanted to write the equation again, but this time writing it with the number that 'x' is representing, we'd write a '2' where the 'x' is now:

$$\Rightarrow x + 7 = 9$$

$$\Rightarrow 2 + 7 = 9$$

Notice how we've ended up with our original, all number equation.

In this last example we used a letter ('x') as a variable. Variables can also be symbols. People sometimes use Greek Symbols as variables, such as θ, called 'theta'. If we used θ instead of 'x' in the last equation, we'd write:

$$\Rightarrow \theta + 7 = 9$$

This time the '2' would be represented by θ.

Now, what happens if we start with an equation which already has a *variable* in it:

$$5 - x = 2$$

This equation has the variable 'x' in it. This time though, we don't already know what number the 'x' is representing. If we want to find out what 'x' represents, we need to "*solve the equation for x*". This means – work out what value the 'x' is representing.

To solve the equation for x, we need to work out what number we can replace the 'x' with to make the equation *true*. So, we might as well try a number – let's try '1':

$$5 - 1 = 2?$$

$$4 \neq 2$$

So instead of writing 'x', I wrote '1' down, and then calculated what 5 – 1 equals – it equals 4. We are trying to get 2 as our answer, *not* 4, so using 1 *does not* make the equation true.

Let's try replacing 'x' with '2':

$$5 - 2 = 2?$$

$$3 \neq 2$$

So using '2' instead of 'x' doesn't make the equation true either. When we put '2' instead of 'x', we get an answer of 3, and we want an answer of 2. But we're on the right track, cause 3 is only just bigger than 2. Let's try replacing 'x' with '3':

$$5 - 3 = 2?$$

$$2 = 2$$

Bingo! When we use '3' to replace 'x', we get an answer of '2', which is what we want. We can say something like: "When x equals three, the equation is true." If you were writing an answer to the question, "Solve the equation for x," you'd write:

$$x = 3$$

Coefficients of variables

A coefficient is the number in front of a variable. Look at the following expression:

$$5x$$

The number in front of the 'x' is its coefficient: 5. What about:

$$-4x+3y$$

The coefficient of 'x' is '–4' and the coefficient of y is '3'.

When I write something like $5x$, all I'm saying is that I have "five lots of x". So if I have an equation like:

$$3x+4x$$

I can read this equation as "Three lots of x plus four lots of x". What happens when I have 3 of something, and I add another 4 of the same thing? Simple – I end up with 7 of that thing. So in this case, I can rewrite this expression:

$$\begin{aligned}&\Rightarrow 3x+4x\\&=7x\end{aligned}$$

What about if I have something like:

$$6x+3y$$

Can I add these two together? The answer is NO. I can only add together variables that are the same. 'x' and 'y' are *different* variables, so I can't add them together.

Multiplying and dividing variables

You can do other things with variables apart from add or subtract them. For instance, variables can be multiplied and divided – say I wanted to multiply 'x' by 'y'. I'd write this as:

$$xy$$

When you're multiplying variables together, often the multiplication symbol isn't written. The last expression is exactly the same as:

$$x\times y$$

except it doesn't have the multiplication symbol.

You can also divide variables by other variables:

$$\frac{x}{y} \text{ or } {}^{x}\!/\!_{y} \text{ or } x/y \text{ or } x\div y$$

are all ways of writing "x divided by y".

Expressions, equations and terms

Some students (such as me) have problems remembering the difference between an *expression* and an *equation*. To understand the difference, you also need to know what a *term* is. A term is part of an equation which has one or more variables or pronumerals in it, but no '+' signs or '–' signs in it. For instance:

- $2ab$ is a term because it has two pronumerals in it, *a* and *b*.
- $\frac{42x^2y^3}{\sqrt{43xy}}$ is also a *term* because it has the two pronumerals *x* and *y* in it, but no '+' or '–' signs in it.
- $(4x+3y)$ is *not* a term because it has a '+' sign in it. In fact, it is made up of *two* terms, '4x' and '3y'.

What about something like '5' – is this a term? Numbers by themselves are often called *numeric* terms – so for something like: $3x+5$, there are two terms – one algebraic or normal term (3x), and one numeric term (5).

So now that we know what a term is, we can understand what expressions and equations are.

An *expression* is a group of terms connected by '+' or '–' signs. There can be as many terms as you want, as long as there is at least 1. For instance,

$$3x^2+2x-15.5$$

is an expression, because it contains three terms, which are joined together by a '+' and a '–' sign.

Expressions *do not have* '=' signs in them. Equations do however. Now that we understand what expressions are, we can use them to help understand what an equation is.

An *equation* is made up of two expressions with an '=' sign between them. One expression is on the left of the '=' sign, and the other expression is on the right of the '=' sign. Here's one example of an equation:

$$3x+4=5x-2$$

Notice that the equation is made up of two expressions – '3x + 4' and '5x – 2'. Also see how they are joined together by an '=' sign. Equations tell the reader that all the stuff on the left of the '=' sign is the same as all the stuff on the right of the '=' sign.

Like terms

Like terms contain the same variables and numbers, raised to the same powers. For instance,

$$3xy \text{ is the same as } 3yx$$

because both terms contain the same number and the same variables. What about:

$$3x^2y \text{ and } 3y^2x\,???$$

These *are not like terms*. They both have the same number – '3'. They also have the same variables – 'x' and 'y'. *However*, the variables are raised to different powers in each

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term. The first term has ‘x’ raised to the 2nd power, but not in the second term. Also, the second term has ‘y’ raised to the 2nd power, but this doesn’t happen in the first term.

Binomial and trinomial expressions

There are two common types of expressions, called *binomial* and *trinomial* expressions. You can guess what these expressions mean by looking at their names. The ‘bi’ part of binomial means ‘two’ – binomial expressions have *two* terms. Now look at the ‘tri’ – tri means ‘three’ – trinomial expressions have *three* terms. So here’s an example of each type of expression:

$3r - 2t$ is a binomial expression.

$2j^2 - 15b^3 + 3a$ is a trinomial expression.

SECTION 3.2 - ALGEBRAIC ADDITION AND SUBTRACTION

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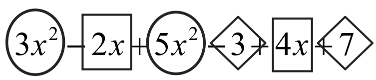
Some of the first questions you get when you start algebra are to simplify expressions. For instance, say you had to simplify the following expression:

$$3x^2 - 2x + 5x^2 - 3 + 4x + 7$$

So, when you look at this expression, one thing you can do straight away is work out that there are 6 terms. To simplify the equation, you need to search for *like terms*:



In this expression there are 3 sets of like terms. The first set contains terms with an ‘x^2’ in them – these have a circle around them. The second set of like terms have only ‘x’ in them – these have a rectangle around them. The last set of like terms are just the numeric terms – these have a diamond shape around them.

Since the only operations in this expression are additions and subtractions, we can *re-order* the terms. By re-ordering the terms, we can have like terms next to each other, which makes the expression a lot easier to simplify:

$$\text{So } 3x^2 - 2x + 5x^2 - 3 + 4x + 7$$

becomes

$$3x^2 + 5x^2 - 2x + 4x - 3 + 7$$

and now we have each set of like terms together – the ‘x^2’ terms are at the start, the ‘x’ terms are in the middle, and the numeric terms are at the end. Now to simplify the expression, we just need to add or subtract the like terms:

$$3x^2 + 5x^2 = 8x^2$$
$$-2x + 4x = 2x$$
$$-3 + 7 = 4$$

So the overall expression becomes:

$$8x^2 + 2x + 4$$

Notice how there are no like terms in this expression anymore – each term is different. This is a good way to check you have done as much simplification as possible. If there are still like terms in the expression you haven't finished simplifying.

SECTION 3.3 - MULTIPLICATION AND DIVISION IN ALGEBRA

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Multiplying and dividing algebraic variables can be confusing when you first start algebra. One of the things you must always remember is that when you have something like:

$$xy$$

this is really

$$x \times y$$

It's just that we often don't write the multiplication symbol in when we're writing algebraic expressions.

Another thing it is important to remember is that:

$$x^2 = x \times x$$
$$x^3 = x \times x \times x$$

When you have a variable raised to the power of 2 or 3, it is simply a shorter way of writing all the multiplication symbols in.

So say we have a question like:

$$2xy^3 \times 4x^2 y$$

Unlike addition and subtraction, to perform multiplication and division you *do not need* like terms. Instead, you are looking to combine variables together. So, first of all, I can rewrite this expression, but separating out all the different bits:

$$2 \times x \times y^3 \times 4 \times x^2 \times y$$

I can further rewrite this expression by putting all the similar variables together:

$$2 \times 4 \times x \times x^2 \times y^3 \times y$$

Now I have all the numbers at the front, the 'x' parts in the middle and the 'y' parts at the end. I can start to simplify the expression by combining similar parts. Firstly, I can see a '2' and a '4' multiplied by each other – I know that $2\times4=8$ so the expression becomes:

$$8\times x\times x^2\times y^3\times y$$

Next, I can deal with the $x\times x^2$ part – remember that $x^2=x\times x$, so this is really just $x\times x\times x$, which is the same as x^3. So the overall expression becomes:

$$8\times x^3\times y^3\times y$$

The last bit I have to deal with is the $y^3\times y$ bit. Another way of writing this would be $y^3\times y^1$. When you multiply two of the same variables together, you can just add their powers (the little numbers above the right hand side of the number). So $y^3\times y^1=y^{3+1}=y^4$. So now the overall expression becomes:

$$8\times x^3\times y^4$$

The last step is to remove the multiplication symbols, since they are not usually written in this type of algebraic expression:

$$8x^3y^4$$

and voila! There's your answer.

Now, how about some expressions that involve division as well:

$$8d^3k^4\div4dk^3$$

I have often found it easier to rewrite this using a fraction instead of a '÷' sign, like this:

$$\frac{8d^3k^4}{4dk^3}$$

This way you can easily separate the expression into parts containing the same variables:

$$\frac{8}{4}\times\frac{d^3}{d}\times\frac{k^4}{k^3}$$

Now, the first part of this expression is easy to simplify – it's just 8 divided by 4, which we all know is 2. Our overall expression becomes:

$$2\times\frac{d^3}{d}\times\frac{k^4}{k^3}$$

Next we have to look at the d^3 divided by d part. The easiest way to do this is to remember that $d=d^1$, so it's really:

$$\frac{d^3}{d^1}$$

When you divide two of the same variables, you can just subtract the 2nd power from the first one:

$$\frac{d^3}{d^1} = d^3 \div d^1 = d^{3-1} = d^2$$

So the overall expression becomes:

$$2 \times d^2 \times \frac{k^4}{k^3}$$

The last bit to do is simply the part with the 'k's in it. This is easy to do:

$$\frac{k^4}{k^3} = k^4 \div k^3 = k^{4-3} = k^1 = k$$

So the overall expression becomes:

$$2 \times d^2 \times k$$

Remove the multiplication symbols, because they're not usually written, and we get:

$$2d^2k$$

Brackets in algebra

Brackets are used in algebra to show that some type of operation needs to be done on *a number* of terms, rather than just one. For instance, if I want to multiply 3x by 4, I just go:

$$3x \times 4$$

But what if I need to multiply $3x - 4y + 7$ by 2? I could go:

$$2 \times 3x - 2 \times 4y + 2 \times 7$$

but this is a bit awkward and looks messy. A much neater way to write it is to use brackets:

$$2(3x - 4y + 7)$$

This expression tells me that everything in the brackets needs to be multiplied by 2.

Factors in algebra

Numbers can have factors. Factors can be multiplied together to give the number. For instance, the factors of 10 are 1, 10, 2, and 5. This is because you can get 10 from them by doing:

$$10 = 1 \times 10$$

$$10 = 5 \times 2$$

But you can also have factors in algebra. Take the following expression for instance:

$$4x^2y$$

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If I read this out aloud, it would sound something like, "four times x squared times y." I can also split this expression up into smaller bits like this:

$$\Rightarrow 4x^2y$$
$$= 4 \times x^2 \times y$$
$$= 4 \times x \times x \times y$$

By splitting it up, what I have done is work out what all its factors are. When we're using numbers, factors multiply together to give the final number. When we're using algebraic variables, factors multiply together to give the final algebraic expression. So the factors of $4x^2y$ are $4, x, y$.

Highest common factor in algebra

When you have two algebraic expressions, the highest factor of *both* expressions is called the *highest common factor*, or *greatest common factor*. Say I had the following expressions:

$$4x^2y \text{ and } 6x^3y^2z$$

To find the highest common factors I like to go through each expression bit by bit. So first, I compare the two number parts – the '4' and the '6'. The highest common factor of 4 and 6 is 2, so I write down 2:

HCF (Highest Common Factor) = 2…

The '…' after the 2 means I haven't finished writing down what the HCF is. Next, I move on to the next bit – the 'x' part. In the first expression, we have a 'x^2', in the second expression we have a 'x^3'. The highest common factors of these two bits is 'x^2'. So I write that down after the '2':

HCF (Highest Common Factor) = $2x^2$…

Now we can move on to the next bit of the two expressions – the 'y' bit. The first expression has a 'y', the second expression has a 'y^2'. This means the HCF of this part is simply 'y'. I can write that down in my answer:

HCF (Highest Common Factor) = $2x^2y$…

The last bit of the expressions I look at is the 'z' bit. However, z is only in the second expression, not the first. This means there is no common factor between the two expressions for 'z'. So I can't write down anything more. This means my answer is:

HCF = $2x^2y$

If you've got time, it pays to check whether this answer makes sense. You can do this by trying to divide both expressions by the HCF, and seeing if you can get an answer. Let's do that now:

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$$\Rightarrow 4x^2y \div 2x^2y \qquad \Rightarrow 6x^3y^2z \div 2x^2y$$
$$= \frac{4x^2y}{2x^2y} \qquad = \frac{6x^3y^2z}{2x^2y}$$
$$= 2 \times 1 \times 1 \qquad = 3 \times x \times y \times z$$
$$= 2 \qquad = 3xyz$$

Two things to look for here. First of all, you should be able to do the division without getting any fractions or decimals in your answer. That checks out in this case. The second thing is to look to see if there are any more common factors between your two answers. If there are, you need to multiply your original answer by that common factor. In this case, there are no common factors for '2' and '3xyz', so it looks like our answer is correct.

Factorising algebra - using common factors to introduce brackets

Brackets can be used to make algebraic expressions much more neat looking and easier to work with. Take the following expression for instance:

$$15q^2j^3 - 5q^3j$$

Looking at this expression, you can see straightaway that there are a lot of common factors – there are 'q's and 'j's in both expressions for instance. You can *factorise* this expression by finding the highest common factor and putting the expression into *factorised form*. Watch this:

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Handy Hint #10 - Introducing brackets in algebra

Find the highest common factor:

For the numbers part, the HCF is 5.

For the 'q's, the HCF is q^2.

For the 'j' part the HCF is j.

So the overall HCF is $5q^2j$.

Take this HCF and put it outside a pair of brackets with some space between them:

$$5q^2j(\qquad\qquad)$$

Now we need to write something inside the brackets. The thing to write is the original expression, *divided by* the HCF like this:

$$5q^2 j\left(\frac{15q^2 j^3 - 5q^3 j}{5q^2 j}\right)$$

Now look at this expression. If we multiplied the bottom of the fraction by the factor outside the brackets, we'd end up with our original expression. But since we're *factorising*, let's just simplify what's inside the brackets:

$$\begin{aligned}&\Rightarrow 5q^2 j\left(\frac{15q^2 j^3}{5q^2 j} - \frac{5q^3 j}{5q^2 j}\right)\\&= 5q^2 j(3\times 1\times j^2 - 1\times q\times 1)\\&= 5q^2 j(3j^2 - q)\end{aligned}$$

If you've got heaps of spare time, you can multiply out the brackets to check you get back to your original expression in *expanded form*. It is useful to be able to switch between expanded form and factorised form quickly and easily.

$$15q^2 j^3 - 5q^3 j \;\underset{\textbf{expand}}{\overset{\textbf{factorise}}{\rightleftarrows}}\; 5q^2 j(3j^2 - q)$$

Expanded form **Factorised form**

So let's try a reasonably complex algebraic simplification problem:

Complex algebraic simplification problem

Simplify $\dfrac{15ab(a^2b - ab)}{5b^2a}$

Solution

So this expression has brackets, multiplication, division, subtraction and addition in it. Brackets are the first operation we need to do, so we look at them first:

$$(a^2b - ab)$$

Can we simplify what is in the brackets? Well, there are two terms, the 2nd term (ab) is being subtracted from the first term (a^2b). Now we can only do the subtraction if they are *like* terms. Like terms have to have the *same variables* raised to the *same powers*. Both these terms have the variables 'a' and 'b', so that's ok. *But* a is raised to different powers in each term, so they are not like terms. This means we can't simplify what's within the brackets.

So now we can look at the whole expression and spread it out:

$$\Rightarrow \frac{15ab\left(a^2b - ab\right)}{5b^2a}$$
$$= \frac{15ab}{5b^2a} \times \frac{\left(a^2b - ab\right)}{1}$$
$$= \frac{15}{5} \times \frac{ab}{ab^2} \times \frac{\left(a^2b - ab\right)}{1}$$
$$= \frac{15}{5} \times \frac{a}{a} \times \frac{b}{b^2} \times \frac{\left(a^2b - ab\right)}{1}$$

In each step I've just done a little bit of re-arranging, trying to separate the expression into bits which only have one variable in them. Now I can simplify each bit at a time:

$$\Rightarrow \frac{15}{5} \times \frac{a}{a} \times \frac{b}{b^2} \times \frac{\left(a^2b - ab\right)}{1}$$
$$= 3 \times 1 \times \frac{1}{b} \times \frac{\left(a^2b - ab\right)}{1}$$

The $\frac{15}{5}$ becomes 3 and the $\frac{a}{a}$ becomes 1. The $\frac{b}{b^2}$ is the same as $b \div b^2$, which if you remember is:

$$b \div b^2 = b^1 \div b^2 = b^{1-2} = b^{-1}$$

Any variable or number raised to a negative power is the same as *1 on that* variable raised to the positive of that power:

$$b^{-1} = \frac{1}{b^{+1}}$$

So the whole expression becomes:

$$\frac{3}{b} \times \frac{\left(a^2b - ab\right)}{1}$$

Now we have to multiply out the brackets. Firstly, we know that we have to multiply *each term in the brackets* by $\frac{3}{b}$. So we can rewrite the expression to show this:

$$\Rightarrow \frac{3}{b} \times \frac{(a^2b - ab)}{1}$$

$$= \frac{3}{b} \times \left(\frac{a^2b}{1} - \frac{ab}{1} \right)$$

$$= \frac{3}{b} \times \frac{a^2b}{1} + \frac{3}{b} \times \frac{-ab}{1}$$

Now there are two multiplication operations we have to do. To multiply fractions we multiply the tops together and the bottoms together. The first one is:

$$\frac{3}{b} \times \frac{a^2b}{1}$$

$$= \frac{3a^2b}{b}$$

$$= \frac{3a^2\cancel{b}}{\cancel{b}}$$

$$= 3a^2$$

The second multiplication is:

$$\frac{3}{b} \times \frac{-ab}{1}$$

Making sure we use the '−' sign :

$$= \frac{-3a\cancel{b}}{\cancel{b}}$$

$$= -3a$$

So overall we have:

$$3a^2 - 3a$$

This can be made a little bit more elegant by using more brackets, although it's more a matter of taste:

$$3a(a-1)$$

SECTION 3.4 - FRACTIONS IN ALGEBRA

Fractions don't have to have only numbers in them. They can also have algebraic variables in them as well. And you also need to be able to add, subtract, multiply and divide *algebraic* fractions together just like you do with normal number only fractions. However, the rules are the same as for working with number fractions.

Adding algebraic fractions

Algebraic fraction addition problem

Calculate $\frac{5x}{2b}+\frac{3x}{4b}$

Solution

First thing we have to is get the two fractions to have a *common denominator* (i.e. have the same thing on the bottom as each other). We can do this by multiplying the tops and bottoms of the left fraction by 2:

$$\Rightarrow \frac{5x}{2b}\times\frac{2}{2}+\frac{3x}{4b}$$
$$=\frac{10x}{4b}+\frac{3x}{4b}$$

Now we've got a common denominator, we can do the rest of the calculation:

$$\Rightarrow \frac{10x}{4b}+\frac{3x}{4b}$$
$$=\frac{10x+3x}{4b}$$
$$=\frac{13x}{4b}$$

Subtraction is just like addition – you need to make sure all the fractions have a common denominator. Now for multiplication and division.

Multiplying algebraic fractions

Multiplying algebraic fractions question

Calculate $\frac{4a}{3b}\times\frac{3b^2}{2a^2}$

Solution

Multiplication just means we have to multiply the tops by the tops (the numerators), and the bottoms by the bottoms (the denominators):

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$$\Rightarrow \frac{4a}{3b} \times \frac{3b^2}{2a^2}$$
$$= \frac{12ab^2}{6a^2b}$$

We also need to simplify it by cancelling out bits.

- 12 divided 6 becomes 2 on the top
- 'a' divided by 'a^2' becomes just 'a' on the bottom.
- 'b^2' divided by 'b' becomes just 'b' on the top.

$$\Rightarrow \frac{12ab^2}{6a^2b}$$
$$= \frac{12}{6} \times \frac{a}{a^2} \times \frac{b^2}{b}$$
$$= \frac{2}{1} \times \frac{1}{a} \times \frac{b}{1}$$
$$= \frac{2b}{a}$$

Dividing algebraic fractions

Dividing algebraic fractions question

Calculate $\frac{2r^2}{5h^3} \div \frac{4r}{10h^2}$.

Solution

The trick to dividing is to change it to a multiplication, by swapping the top and bottom of the fraction *that is doing the dividing*. In this case, the $\frac{2r^2}{5h^3}$ is the fraction that *is being divided*, and $\frac{4r}{10h^2}$ is the fraction *that is doing the dividing*. So we can turn this into a multiplication by swapping the tops and bottoms of the $\frac{4r}{10h^2}$:

$$\Rightarrow \frac{2r^2}{5h^3} \div \frac{4r}{10h^2}$$
$$= \frac{2r^2}{5h^3} \times \frac{10h^2}{4r}$$

Now we can just calculate an answer like a normal multiplication:

$$\Rightarrow \frac{2r^2}{5h^3} \times \frac{10h^2}{4r}$$
$$= \frac{20r^2h^2}{20rh^3}$$
$$= \frac{20}{20} \times \frac{r^2}{r} \times \frac{h^2}{h^3}$$
$$= \frac{1}{1} \times \frac{r}{1} \times \frac{1}{h}$$
$$= \frac{r}{h}$$

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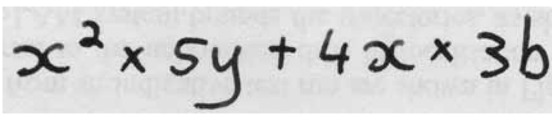
Handy Hint #11 - Confusing 'x' and '×'

This is one of the most common mistakes made in exams – when students don't notice the difference between a multiplication operation (×) and an 'x' symbol. When you're doing algebraic problems you want to make sure that these two things look *really, really different*. And remember, even though *you* may be able to tell the difference, what about your teacher, who may be marking your exam late at night in bad lighting conditions when their eyes are tired. It's best to play safe. For instance, I draw my 'x's as if they were made up of two 'c's put together, with one of the 'c's back to front. My multiplication symbols are smaller and made up of two straight lines, like this:



You can use whatever style you want, as long as it is clear to anyone reading it what the difference is between the two symbols.

SECTION 3.5 - TRANSLATING PROBLEMS INTO ALGEBRA

One of the most important things you need to be able to do is work out how you can use algebra to solve a problem that you are given. Students often struggle understanding and interpreting a written problem and rewriting it in terms of algebraic variables. To give you an idea of what to do, we'll do a few sample problems.

Algebraic translation question

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John walked a certain distance in the morning, before he had lunch. Then in the afternoon, John walked twice as far as he had in the morning. In total that day John walked 27 km. How far did John walk in the morning?

Solution

First things first. We can say with a fair amount of confidence that the solution to this problem will involve algebra, as the question is in the algebra section of this book. The general trick with these questions is to try and *assign* algebraic variables to *quantities* or *amounts* in the question. This will often get you started.

So in this question what quantities or amounts are there. Well there's John, but he's not really a quantity – he's just a person. What about lunch – well that's just a meal, not really an amount or a quantity. What about the distance John walked – bingo! The distance he walked is a quantity – it's some number of kilometres.

Now, in general, you need to *assign* variables to represent quantities that you *don't know anything about*. So, let's think about distances. John walked a certain distance in the morning. He also walked another distance in the afternoon. And then there's the total distance he walked during the whole day. However, we know this last amount – he walked *27 km* during the whole day. So the unknown amounts are how far he walked in the morning and how far he walked in the afternoon.

So, what we can do is *assign* an algebraic variable to represent one of these unknown quantities – let's start with the distance that John walked in the morning, since this is the amount we're trying to find out:

Let x = the distance that John walked in the morning

So now we've written a simple sentence telling the reader that 'x' now represents the distance that John walked in the morning.

Another thing that will help you get started in many algebraic problems is to look for *relationships* between quantities in the question. In this question for instance, there is a relationship between the distance John walked in the morning and the distance he walked in the afternoon. He walked *twice as far* in the afternoon as he did in the morning. We can add this to our solution:

$$\text{Distance walked in afternoon} = 2 \times \text{distance walked in morning}$$

$$\text{Distance walked in afternoon} = 2 \times x$$

$$\text{Distance walked in afternoon} = 2x$$

This should make sense – in the afternoon he walked twice as far, so he walked a distance of '2x' in the afternoon, compared with only 'x' in the morning.

So, we still need to find out how many kilometres John walked in the morning – in other words we have to find out what 'x' equals. We need to write an *algebraic equation relating* 'x' to some number. The only number in this question we have been given is the total number of kilometres John walked in the day – 27 km.

We already have algebraic expressions for the distance John walked in the morning and afternoon – we can join these together to give the total distance he walked in the day:

$$\text{Distance in morning} + \text{distance in afternoon} = \text{total distance}$$

$$x + 2x = \text{total distance}$$

$$3x = \text{total distance}$$

$$3x = 27 \text{ km}$$

$$x = 9 \text{ km}$$

Now all we have to remember is to write the answer at the end of our solution:

Answer: John walked 9 km in the morning.

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